

Optimum design of one way concrete slabs cast against Textile Reinforced Concrete Stay-in-Place Formwork Elements*

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Summary: This study presents a conceptual design process for one-way reinforced concrete slabs cast over Textile Reinforced Concrete (TRC) Stay-in-Place (SiP) formwork elements, aiming at the minimization of the composite slab cost satisfying Ultimate Limit State (ULS) and Serviceability Limit State (SLS) design criteria. The thin-walled TRC element is considered to participate in the structural behaviour of the composite slab. This distinct function of the TRC element (as formwork and as a part of a composite element) distinguishes the design procedure into two States: a Temporary and a Permanent one. Design parameters such as the type of the textile reinforcement (material), the geometry of the TRC cross-section, the flexural strength of the fine-grained concrete in the TRC element and the compressive strength of the cast in-situ concrete are considered as the main optimization variables.

1 Introduction

1.1 TRC Stay-in-Place formwork elements

The use of TRC Stay-in-Place formwork elements entails economical, mechanical, aesthetic and structural benefits; namely, ready-to-use surfaces, reduced construction time, elimination of falsework, high load-bearing capacity and durability. Previous works REINHARDT [1], BRAMESHUBER ET AL. [2] examined two basic types of cementitious Stay-in-Place formwork elements and introduced the basic principles for this type of elements. This study outlines an optimum (minimum cost) design procedure for permanent TRC formwork elements taking into consideration Ultimate Limit State and Serviceability Limit State criteria.

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1.2 Design Considerations

The design procedure should be distinguished into a Temporary State (TS) and a Permanent one (Fig. 1). The Temporary State corresponds to the time period following the construction of the formwork and includes all possible loading conditions that might be imposed on it due to handling, storing, transportation and in situ assembling. In the TS only the thin-walled element is considered, whereas in the Permanent State (PS) the composite slab (TRC + in-situ concrete topping) is examined. For the Temporary State the critical design load is the uniform load corresponding to the sum of the self-weights of the TRC elements, the steel reinforcement and the freshly poured concrete. According to ACI 347 [3] the design load for combined dead and live uniform load in TS is $4.8 \times 10^{-3} \text{ N/mm}^2$, but also other load cases have to be considered, such as impact loading (e.g. accidental dropping of tools) and high concentrated loads (e.g. stacking of supplies). The Permanent State corresponds to the time period following the hardening of the cast in-situ concrete. Design loads for the composite slab are defined in EN 1991 [4].

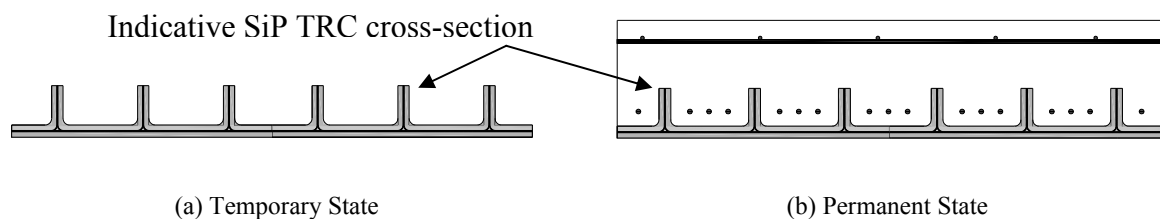


Fig. 1: Design States

For high support spacing of the TRC element (up to 6 m), the design bending moment is probable to exceed the cracking bending moment. In this case the designer has to decide if cracking formation is acceptable in this state of design. If cracking is not acceptable an intermediate support must be used (increasing the construction cost). Because of the inclusion of a propping system the structural system is also changed (from simply supported beam to continuous beam) and tensile stresses in the upper part of the element are introduced; this calls for the addition of textile reinforcement (or other) into the regions under tension. The use of open-type sections in this case is not recommended.

Assuming that crack formation in TS is allowed, the crack width must be limited. The limiting value of the crack width corresponds to a fraction (k_w) of the maximum allowed crack width for reinforced concrete slabs. The use of fine-grained concrete (for the TRC element) permits the adoption of a maximum crack width equal to 0.3 mm, a value larger than the one allowed for conventional concrete slabs, although crack widths in TRC elements usually range between 0.05 and 0.2mm. Furthermore, the deflection of the formwork element must not exceed a maximum value which may be taken equal to a percentage (k_d) of the total al-

lowed deflection for the composite slab (equal to $l/250$). Coefficients k_w and k_d determine the desirable degree of contribution of the TRC element to the composite structure.

In the Permanent State the design must ensure the adequate response of the composite element against all possible failure modes within the mandate of the Ultimate Limit State. All limitations dictated by the Serviceability Limit State must also be satisfied. The desirable failure mode is the excess of maximum compressive strain in the top concrete fibre. Table 1 summarises all necessary checks for the design of TRC SiP formwork elements for the construction of one-way reinforced concrete slabs.

Table 1: Calculations for the design states

Temporary		Permanent	
SLS	ULS	SLS	ULS
Deflections	Bending	Deflections	Bending
Crack width	Vertical Shear	Crack width	Vertical Shear
<i>Impact loading</i>		Vibrations	Horizontal Shear
<i>Differential shrinkage</i>		<i>Differential shrinkage</i>	

2 Mechanical behaviour-Design Equations

2.1 Resistance capacity of the TRC Element (TS)

Flexural strength can be computed through a cross-section analysis by implementing a fibre model for the sake of generality of programming and accuracy. It has been shown by JESSE ET AL. [5] that the use of textile layers comprising un-coated fibre rovings results in the reduced contribution of such reinforcement to the TRC's flexural strength. This phenomenon is attributed to the limited penetration of the cementitious matrix in the fibre rovings, which in turn reduces the fibre-to-fibre bond.

It should be underlined that fibre-matrix interaction is a poly-parametric problem relying on a large number of factors. The use of cementitious matrices (a per se composite material) introduces a higher degree of complexity in understanding and adequately modelling this interaction; matrix compositions, filament surface treatments, types of rovings' coatings used (see also MÄDER [6], KEIL [7]) result in an infinite number of possible combinations rendering the derivation of a global micromechanical model even more difficult. Significant experimental and analytical work in bond mechanisms in TRC has been conducted by KRÜGER [8, 9, 10], BANHOLZER [11], CHUDOKA [12], JESSE [5], to mention a few. A combination of the micro- and mesoscopic models leading to the so-called Micro-Meso-Macro-Prediction Model (MMM-PM) has been proposed by LEPENIES [13]. In this work, and in sake of generalization

and simplification that should characterize any design tool, textile-to-matrix interaction is considered to be primarily dependent on the rovings' coating (or its absence thereof).

In order to take into account the effect of roving's coating in a cross-section analysis, two coefficients are introduced. The first, k_b , quantifies the part of the textile reinforcement area that is in good contact with the cementitious matrix and, therefore, exhibits high bond conditions with the latter, and the second, k_s , accounts for the 'strain lag' between inner and outer filaments in uncoated rovings.

More specifically, for textiles with coated rovings coefficients k_b and k_s are taken equal to unity denoting full resin impregnation of the fibre bundles, perfect bond of the textile to the cementitious matrix and uniform strain profile in each fibre roving. For textiles with uncoated (according to PAPANICOLAOU ET AL. [14]) rovings coefficients k_b and k_s are taken equal to 0.25 and 0.4, respectively, denoting that perfect filament-to-matrix bond conditions (attributed to mortar penetration in the rovings) are achieved within only a fraction (1/4) of the textile reinforcement area (located at the periphery of the rovings in practice) and that during loading the 'core' fibre reinforcement is under lower strain conditions compared to the reinforcement comprising the outer well-bonded 'sleeve' (Fig. 2). The proposed coefficients are to be seen as "box" values, which may be altered in the future, should more experimental data be systemically gathered and organized. The above considerations originate from the ideas included in the 'adhesive cross linkage model' SCHORN [15] and the 'plug model' LEPENIES [16].

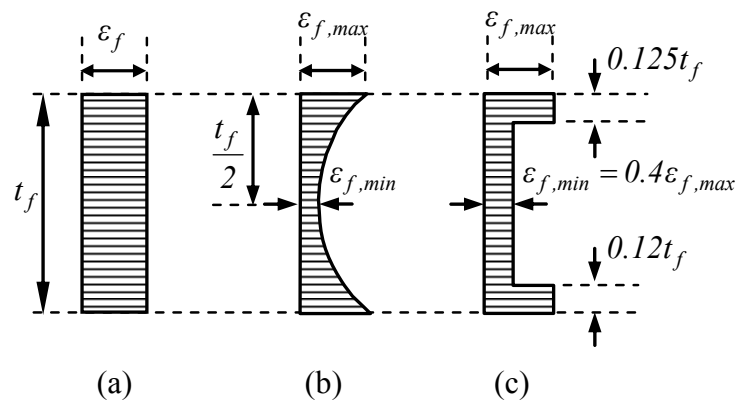


Fig. 2: Idealization of the strain profile across the cross-section of a fibre roving: (a) Uniform strain distribution in a coated fibre roving; (b) Actual strain distribution in an un-coated fibre roving; and (c) Simplified approximation of strain distribution in an un-coated fibre roving

Applying these two coefficients resistance capacity of the textile reinforcement can be computed by Equation 1

$$F_{tex} = k_b A_f E_f \varepsilon_{f,max} + (1 - k_b) A_f E_f (k_s \varepsilon_{f,max}) \quad (1)$$

with F_{tex} Resistance capacity of the textile reinforcement
 A_f Total textile reinforcement area
 E_f Modulus of Elasticity of fibrous material
 $\varepsilon_{f,max}$ Maximum strain (i.e. strain of sleeve filaments)

Failure strain ε_{fu} is taken equal to 85% of the nominal failure strain of the fibrous material and is computed from Equation 2

$$\varepsilon_{fu} = 0.85 \frac{f_{ft}}{E_f} \quad (2)$$

with f_{ft} Tensile strength of the fibrous material

For the cementitious matrix a parabolic stress strain relation may be adopted according to EN 1992 [17]. Shear strength capacity TRC may be computed according to VOSS ET AL. [18].

2.2 Deflection of the TRC Element

For uniform loading conditions deflections are given from Equation 3

$$\delta_{TRC} = \frac{5}{48} \phi l^2 \quad (3)$$

with ϕ Curvature at mid-span section
 l Span between supports

Curvature can be computed from Equation 4 according to EN 1992 [17].

$$\phi = \left(1 - \beta_1 \beta_2 \left(\frac{M_{cr}}{M_{sd}} \right)^2 \right) \phi_{II} + \beta_1 \beta_2 \left(\frac{M_{cr}}{M_{sd}} \right)^2 \phi_I \quad (4)$$

with	ϕ_I	Curvature of section at first crack
	ϕ_{II}	Curvature of fully cracked section
	β_1	Coefficient taking into account bond conditions between the textile reinforcement and the cementitious matrix (0.5 is a conservative value)
	β_2	Coefficient taking into account the duration of loading (0.5 for permanent loading)
	M_{cr}	Cracking Moment, dependent on the tensile strength of the cementitious matrix
	M_{sd}	Design Moment

2.3 Crack width of the TRC Element

Crack width may be computed from Equation 5

$$w_k = s_m (\varepsilon_{fm} - \varepsilon_{cm}) \quad (5)$$

with	s_m	Maximum crack spacing
	ε_{fm}	Mean strain in the textile reinforcement under the specified load combination
	ε_{cm}	Mean strain in the fine-grained concrete between cracks

and

$$\varepsilon_{fm} - \varepsilon_{cm} = \frac{I}{E_f} \left(\sigma_s - k_t \frac{f_{ctm,fl}}{\rho_{f,eff}} (1 + a_f \rho_{f,eff}) \right) \quad (6)$$

with	a_f	Modular ratio (E_f / E_{matrix})
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$\rho_{f,eff}$	Effective reinforcement ratio (A_f / A_{ct})
σ_s	Stress in textile reinforcement assuming a cracked section
k_t	Factor dependent on the duration of loading
$f_{ctm,fl}$	Mean flexural strength of fine-grained concrete

2.4 Calculations for the composite element (PS)

Calculations for the composite section for flexure and shear follow the rules of Eurocode 2. Treatment of the textile reinforcement is the same as in the previous section. In Serviceability Limit State the maximum deflection and crack width is given by Eq. (7) and (8), respectively.

$$\delta_{comp} \leq \frac{l}{250} - \delta_{TRC} \quad (7)$$

$$w_{comp} \leq 0.3 - w_{TRC} \quad (8)$$

Horizontal shear in the composite element depends on the interface shear capacity of the cast-in-situ concrete and the fine-grained concrete of the TRC element. The formwork's roughness is taken into consideration, whereas no dowels are used to transfer shear stresses between the two structural components of the composite element. The formulations follow the format of EC2.

3 Formulation of optimum design

3.1 Design Parameters

The optimization process is an iterative process aiming at the minimization (in this case) of the objective function, subjected to various constraints. In structural optimization the objective function is usually the total cost function. Hence, the optimum solution is achieved when the cost is minimized and all the constraints are satisfied.

The main criterion for the selection of a design variable is the degree of participation in the total cost of the element. The global design variables can be classified into two categories; the first one includes the parameters that define the section geometry, namely the ratio of the thickness of the TRC element to the effective depth of the textile reinforcement (measuring from the reinforcement layer level to the full height of the composite member) t/d_f , and the

ratio of d_f to the effective depth of the textile reinforcement (measuring from the reinforcement layer level to the height of the TRC element). Local (geometrical) design parameters are assigned for each type of cross-section considered. Figure 3 illustrates some sections for use as a SiP TRC formwork element. The second category of design variables includes the material characteristics (flexural strength of the fine-grained concrete and compressive strength of the cast in-situ concrete) and the reinforcement ratios (textile and steel).

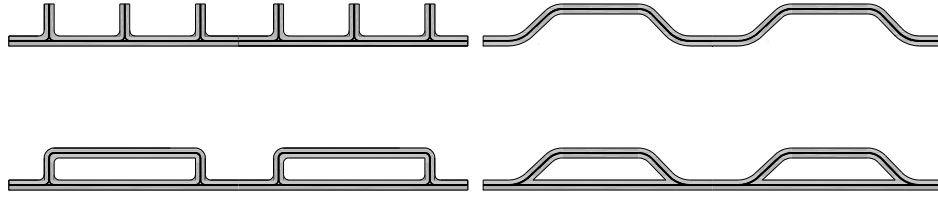


Fig. 3: Probable SiP TRC cross-sections

3.2 Objective function

In an optimization problem one has to decide about the most decisive parameters that influence the objective function (in this case the minimum cost function) and draw a line on the number of important factors considered. It is without doubt that factors such as the manufacturing process of the textile or of the TRC element itself contribute to the total final cost of the composite element. Nevertheless, since the optimum design process mainly addresses the needs of the designer the manufacturing costs are considered as precast plant-dependent and cannot be incorporated in a general optimization design process.

In this design process the total cost of the element is the sum of four individual cost components and is given from the following equation.

$$C = C_c \frac{A_c}{bd_f} + C_m \frac{A_m}{bd_f} + C_s \gamma_s \rho_s \frac{d_s}{d_f} + C_{tex} \gamma_f \rho_f \quad (9)$$

with	C	Total cost per unit volume [i.e. monetary units per ℓbd_f ($\text{€}/\text{m}^3$), where ℓ and b are the element's length and width, respectively]
	C_c	Cost per unit volume of the cast in-situ concrete as a function of its compressive strength ($\text{€}/\text{m}^3$)
	C_m	Cost per unit volume of the fine-grained concrete in the TRC as a function of its flexural strength

C_s	Cost per unit weight of the steel reinforcement (€/gr)
C_{tex}	Cost per unit weight of the textile reinforcement (€/gr)
A_c, A_m	Cast in-situ concrete and fine-grained concrete cross-section areas, respectively
ρ_s, ρ_f	Steel and textile reinforcement ratios (A_s / bd_s & A_f / bd_f), respectively; where A_s steel reinforcement area, A_f textile reinforcement area parallel to ℓ , d_s the effective depth of the steel
γ_s, γ_f	Density of steel and textile reinforcement, respectively

The cost of cast in-situ concrete of normal strength-class (up to C30/37) is usually expressed as a second order polynomial function of its compressive strength. Although the cost of fine-grained concrete is assumed to be a function of mainly the ratios of polymers and fine particles in the mix, in this study (and in sake of simplicity) it is regarded as being dependent solely on the polymer content. Thus, a relation between the flexural strength of the fine-grained concrete and its cost can be established (given in general non-linear form in Figure 4a). The quantification of such a relation is yet to be accomplished. Figure 4b presents a cost vs compressive strength curve for the cast in-situ concrete (based on actual current market values).

The cost of the textile reinforcement is dependent on a large number of parameters such as: fibre material (e.g carbon, glass, aramid, basalt), fibre treatment (sizing, fibre coating), textile geometry (grid spacing, TEX, rovings' stitching material and pattern, etc.). The derivation of a cost function for the textile reinforcement that includes all the above -mentioned parameters does not lie within the scope of the current study and is against the robustness of the optimization process. Table 2 shows the cost relation between textiles considering bi-directional carbon textile reinforcement with un-coated rovings as a reference material. The relative costs of textiles given in Table 2 refer to textiles of equal thicknesses (should smeared distribution of fibres be considered).

3.3 Constraints

The mathematical constraints are posed by the demand that all calculated values derived from the ultimate limit state and serviceability state analyses are lower than or equal to the ones

that cause failure, or are unacceptable in terms of users' comfort. Geometric-specific constraints are also imposed on individual cross-sections due to anthropometric restrictions (that take into account the on-site constructability and site safety).

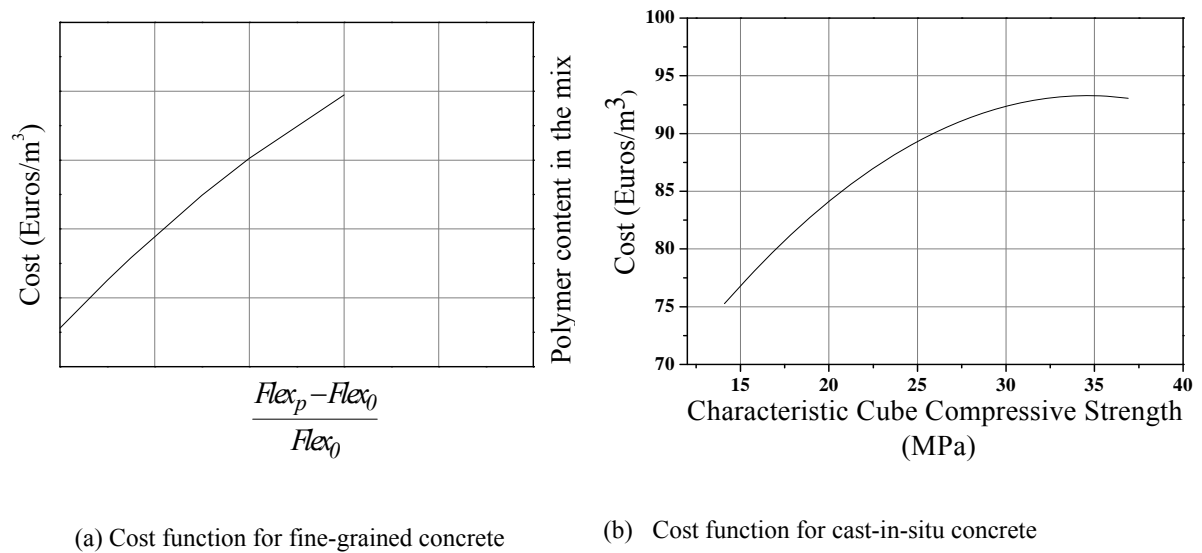


Fig. 4: Cost functions for fine-grained and cast-in-situ concrete

Table 2: Relative costs of different textiles

Coating	No polymer coating			Polymer coating + stitching		
Fibre	Carbon	AR-Glass	Basalt	Carbon	AR-Glass	Basalt
Cost	C_{ref}	$0.65 C_{ref}$	$0.60 C_{ref}$	$1.20 C_{ref}$	$0.78 C_{ref}$	$0.72 C_{ref}$

4 Conclusions

In this study the optimum design of SiP TRC formwork elements and resulting composite reinforced concrete one-way slabs was regarded as a non-linear optimization procedure, which involves the minimization of a cost functions subjected to constraints imposed by specific performance requirements.

The study is on-going and is yet to be supplemented by several runs of the optimization software module and by experimental investigation on large-scale specimens. Before any concrete numerical and experimental results become available (and based on engineering judgment) it could be postulated that textiles balancing between geometrical / productional simplicity and adequate mechanical performance are expected to be preferred over “high-

tech” ones, perhaps even at a slight or moderate expense of economy, as “off-the-shelf” products will find their way to the building industry much faster than the ones specifically engineered for such applications.

In light of the above, stable bi-directional textiles (most probably with resin-impregnated rovings) are to be preferred. Choice between single- and multi-layered TRC systems is strongly connected to the degree of exploitation degree of the reinforcement. For multi-layered systems, if adequate mortar penetration through the textile perforations is ensured for each textile layer then the specific surface of the reinforcement is increased, stress distribution is homogenized and bond is improved. In this case, an adequate mortar thickness in between textile layers must also be provided in order to minimize the “weak planar zones” in the element and the associated risk of inter-laminar shear failure. For practical reasons though, adequate mortar penetration and high mechanical interlock is difficult to be achieved in multi-layered systems. Moreover, constructability constraints of full-scale elements in a precast plant will pose additional restrictions in the maximum number of textile layers used. As a rule of thumb, more than four layers are generally difficult to handle (except when elements are manufactured by compression moulding).

The study aims at shedding some light on “grey areas” that mostly concern the mechanical behaviour of the composite structural member (e.g. crack width, degree of composite action, etc.) and, thus, influence the optimum design of SiP TRC formwork elements. Future work should be focused on supplementing the optimization procedure with durability criteria and long-term mechanical behaviour constraints.

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